Department of Electrical and Computer Engineering
Clemson University

ECE 440/640: Performance Analysis of Local Computer Networks
Fall 2009

Homework #3

Assigned September 15, 2009 **Due Date September 24, 2009**

For all students

1. Consider a discrete random variable K with Poisson distribution and p.m.f.

   $$p_K(k) = \frac{(\lambda t)^k}{k!} e^{-\lambda t}, \text{ k=0, 1, 2, …}$$

   Determine the mean, variance, and standard deviation of K. Hint: Use identities

   $$e^{\lambda t} = \sum_{k=0}^{\infty} \frac{(\lambda t)^k}{k!} \text{ and } \sum_{k=1}^{\infty} \frac{(\lambda t)^k}{(k-1)!} = \sum_{j=0}^{\infty} \frac{(\lambda t)^{j+1}}{j!} = \lambda t e^{\lambda t}.$$

2. Persons arrive at a copy machine in a Poisson arrival process with mean arrival rate 1 per minute. Each person makes a random number of copies – the number of copies is a discrete random variable uniformly distributed between 1 and 10. Each copy takes 3 seconds.
   a. What is the average waiting time in queue for a person if customers are serviced in a FCFS discipline?
   b. Consider persons with no more than 3 copies are given a high priority and are serviced in a non-preemptive discipline. What is the average waiting time in queue for a high priority person?
   c. b. continued, what is the average waiting time in queue for a low priority person?
   d. b. continued, what is the average waiting time in queue among all persons?

3. A communication link of 100 k-bits/sec bandwidth is used to service concentrated traffic from two sources; each source generates Poisson arriving packets with a rate 1000 packets/min. Each source may produce two types of packets: long packets (2500 bits), and short packets (100 bits). Short packets are always given non-preemptive priority over long packets.
   a. If each source has a probability of ½ to produce short packets, what is the utilization of each priority class?
   b. a. continued, if long packets are class 1 and short packets are class 2, find W_1, T_1, N_1 Nq_1, W_2, T_2, N_2, Nq_2, W, T, N, Nq.
   c. Consider a different situation. One source produces only short packets and the other produces only long packets, what is the utilization of each priority class?
   d. If, instead, both long and short packets are assigned the same priority, what is the average waiting time of a packet?

4. A network concentrator with three input links is modeled as a single-server queuing system. The concentrator supports three head-of-line priorities and non-preemptive service. Each input link is assigned a different priority.

Input link A has:	Poisson arrivals at rate 1 per second; exponentially distributed service time with mean 1/5 seconds.
Input link B has:	Poisson arrivals at rate 2 per second; constant service time 1/4 seconds.
Input link C has:	Poisson arrivals at rate 3 per second; exponentially distributed service time with mean 1/15 seconds.

   a. Is the system stable?
   b. What is the average waiting time of a packet from each link, if all packets are given the same priority?
   c. Let class 1 be the lowest priority and class 3 be the highest priority, what is the respective average delay in system for packets from input link A, B, and C, respectively, if their priorities are (A=3, B=2, C=1)?
   d. b. continued, what is the average delay in system for all packets?

<u>For ECE 640 students</u>

5. A batch-scheduling computer system executes computer programs assigned to CPU one by one in FCFS order. Consider programs arriving in Poisson process with rate 3 programs per second. The length of a program is exponentially distributed with an average size of 100 million instructions. The system has three CPU's each with an execution rate of 200 million instructions per second (MIPS). When CPU's are busy, programs wait in one single queue, and the program at the front is assigned to any computer that is available.
   a. What is the probability that three CPU's are busy simultaneously?
   b. What is the average delay in system for a packet?
   c. What is the utilization of each CPU? Note that, when assigning a program to an available CPU, all available CPU's have a uniform probability of getting assigned.

6. Hammond and O'Reilly, Problem 3.16. Hint: use technique from Text Ch.4.4.

7. With the state transition-rate approach in Hammond & O'Reilly Ch.4.4.1, determine the **average number of messages in system** for a M/M/1/∞/P queuing system.